Dymore User's Manual
Unsteady Hydrodynamics

Contents

1 Cable kinematics

Consider a cable idealized as a line in space and an inertial reference frame $\mathcal{I} = (\bar{\imath}_1, \bar{\imath}_2, \bar{\imath}_3)$ is depicted in fig. 1. A point **P** on the cable is defined by its curvilinear coordinate α_1, which measures length along the reference configuration of the cable. The position vector of point **P** is

$$\underline{x} = \underline{x}(\alpha_1). \tag{1}$$

The base vector in the reference configuration, $\bar{g}_1$, is defined as

$$\bar{g}_1 = \frac{\partial \underline{x}}{\partial \alpha_1}, \tag{2}$$

and clearly is the unit vector tangent to the cable in its reference configuration.

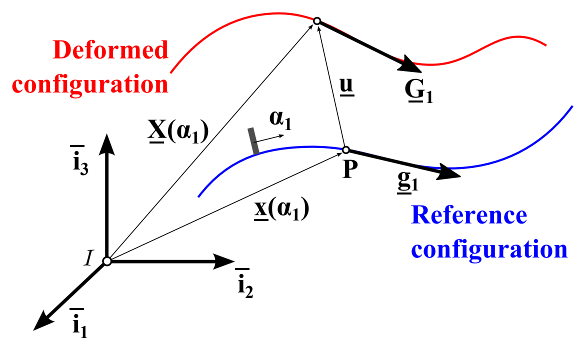

Figure 1: Cable in the reference and deformed configurations

After deformation, the position vector of point **P** is denoted $\underline{X}$ and is written as

$$\underline{X}(\alpha_1) = \underline{x}(\alpha_1) + \underline{u}(\alpha_1), \tag{3}$$

where $\underline{u}$ denote the components of the displacement vector resolved in the inertial basis. The base vector in the deformed configuration becomes

$$\underline{G}_1 = \frac{\partial \underline{X}}{\partial \alpha_1} = \bar{g}_1 + \underline{u}', \tag{4}$$

where notation $(\cdot)'$ indicates a derivative with respect to α_1. Note that $\underline{G}_1$ is tangent to the deformed configuration of the cable, but is not a unit vector.

2 Hydrodynamic forces acting on the cable

The external forces that acting on the cables include the buoyancy force, denoted $\underline{\mathcal{F}}^B$, the hydrodynamic drag force, denoted $\underline{F}^D$, and the inertial force, denoted $\underline{F}^I_a$, due to the added mass effect.

2.1 The buoyancy forces

The volume of the cable per unit length, denoted v_c, is

$$v_c = \frac{\pi D^2}{4}, \tag{5}$$

where D denoted the diameter of the cable. The magnitude of the buoyancy force acting per unit length of the cable equals the weight of fluid that occupies the volume of cable and this force is directed in the opposite direction of the gravity vector, *i.e.*,

$$\underline{\mathcal{F}}^B = -v_c \rho_f \underline{g}, \tag{6}$$

where ρ_f is the mass density of the fluid surrounding the cable and $\underline{g}$ the gravity acceleration vector.

2.2 The drag forces

The formulation of two-dimensional, unsteady hydrodynamic theories typically requires the velocity of the flow with respect to the cable, which simply writes

$$\underline{\hat{v}}_a = \underline{V}_\infty - \underline{\dot{u}}, \tag{7}$$

where $\underline{V}_\infty$ is the far field flow velocity, assumed to be of constant magnitude and orientation and $\underline{u}$ is the displacement field of the cable defined by eq. (3).

The unit tangent vector to the cable in its deformed configuration, see fig. 2, is denoted $\bar{a}_1 = \underline{G}_1/\|\underline{G}_1\|$, where $\underline{G}_1$ is the base vector in the cable's deformed configuration defined by eq. (4). A basis $\mathcal{A} = (\bar{a}_1, \bar{a}_2, \bar{a}_3)$ is attached to the cable; unit vector $\bar{a}_1$ is tangent to the cable and unit vectors $\bar{a}_2$ and $\bar{a}_3$ are in the plane normal to the cable, but their orientations in that plane are arbitrary. The orientation of basis $\mathcal{A}$ with respect to inertial basis $\mathcal{I}$ is defined to rotation tensor $\underline{\underline{R}}_a$.

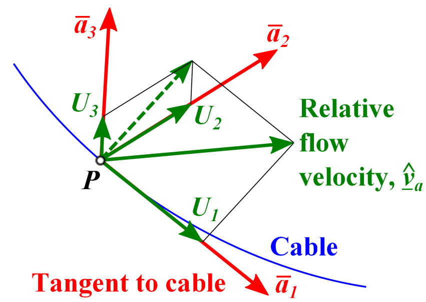

Figure 2: Drag force acting on the cable

As shown in fig. 2, the components of the relative velocity vector resolved in basis $\mathcal{A}$ are denoted

$$\underline{\hat{v}}_a^* = \underline{\underline{R}}_a^T \underline{\hat{v}}_a = \begin{Bmatrix} U_1 \\ U_2 \\ U_3 \end{Bmatrix}, \tag{8}$$

where notation $(\cdot)^*$ indicates components of vectors and tensors resolved in basis $\mathcal{A}$.

Next, the relative acceleration of the flow is evaluated as $\underline{\dot{\hat{v}}}_a = \underline{\dot{V}}_\infty - \underline{\ddot{u}} \approx -\underline{\ddot{u}}$, where the last statement follows assuming the far field flow to be of constant magnitude and orientation. The components of the relative acceleration vector resolved in basis $\mathcal{A}$ are denoted

$$\underline{\hat{a}}_a^* = \underline{\underline{R}}_a^T \underline{\dot{\hat{v}}}_a = \begin{Bmatrix} A_1 \\ A_2 \\ A_3 \end{Bmatrix}. \tag{9}$$

The relative flow velocity is decomposed into its component acting in the plane perpendicular to the cable, denoted $V_\perp = \sqrt{U_2^2 + U_3^2}$ and that acting in the direction tangent to the cable, denoted $V_{||} = U_1$. The drag force per unit length of the cable acting in the plane perpendicular to the cable is

$$F_\perp^d = c_d D \frac{\rho_f V_\perp^2}{2} \left[\cos\alpha \bar{a}_2 + \sin\alpha \bar{a}_3\right], \tag{10}$$

where $U_2 = V_\perp \cos\alpha$, $U_3 = V_\perp \sin\alpha$, and c_d the drag coefficient for the cable. Typically, the drag coefficient is measured from experiment and $c_d \in [0.5, 2.0]$. The drag force per unit length of the cable acting along the cable direction is

$$F_{||}^d = c_f \pi D \frac{\rho_f V_{||}^2}{2}, \tag{11}$$

where c_f is the frictional coefficient of the cable.

In summary, the hydrodynamic model predicts drag forces along axes $\bar{a}_1$, $\bar{a}_2$, and $\bar{a}_3$, denoted F_1, F_2, and F_3, respectively. The expressions for these forces are found from eqs. (11) and (10) as

$$F_1 = \frac{1}{2} c_f \pi D \rho_f V_{||} V_{||}, \tag{12a}$$

$$F_2 = \frac{1}{2} c_d \;\; D \rho_f V_\perp U_2, \tag{12b}$$

$$F_3 = \frac{1}{2} c_d \;\; D \rho_f V_\perp U_3. \tag{12c}$$

2.3 The apparent fluid inertial forces

As the cable moves through the fluid, the surrounding fluid must move as well, creating apparent mass effects. The apparent inertial force $\underline{F}^a$ accounts for such effects

$$\underline{F}^a = (1 + c_a)\rho_f v_c \underline{\dot{V}}_\infty - c_a \rho_f v_c \underline{\ddot{u}}. \tag{13}$$

where c_a is an empirical coefficient such that $c_a \in [0.5, 1.0]$.