The four-bar mechanism

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Figure 1 depicts a flexible four bar mechanism. Bar 1 is of length 0.12 m and is connected to the ground at point **A** by means of a revolute joint. Bar 2 is of length 0.24 m and is connected to bar 1 at point **B** with a revolute joint. Finally, bar 3 is of length 0.12 m and is connected to bar 2 and the ground at points **C** and **D**, respectively, by means of two revolute joints.

In the reference configuration, the bars of this planar mechanism intersect each other at 90 degree angles and the axes of rotation of the revolute joints at points **A**, **B**, and **D** are normal to the plane of the mechanism. The axis of rotation of the revolute joint at point **C** is rotated by +5 degrees about unit vector $\bar{\imath}_2$ indicated in fig. 1 to simulate an initial defect in the mechanism. The angular velocity at point **A** of bar 1 is prescribed to be $\Omega = 0.6$ rad/s.

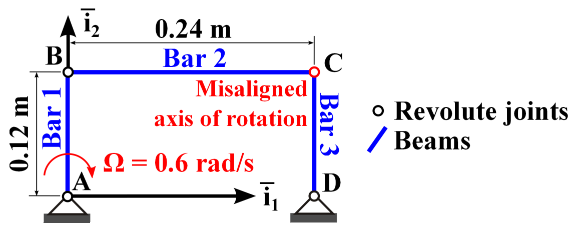

Figure 1: Configuration of the four-bar mechanism.

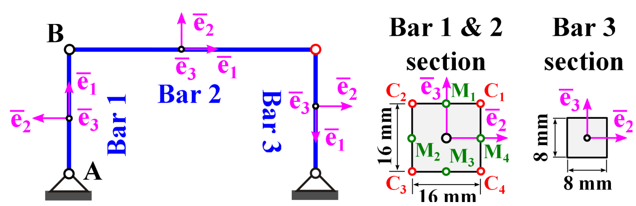

Figure 2: Configuration of the four-bar mechanism.

Figure 2 shows the cross-sections of the bars. Bars 1 and 2 are of square cross-section of size 16 by 16 mm; bar 3 has a square cross-section of size 8 by 8 mm. The three bars are made of steel, whose mechanical characteristics are Young's modulus E = 207 GPa and Poisson's ratio $\nu = 0.3$. These physical properties translate to the sectional stiffness properties listed in table 1. The sectional mass properties are as follows: mass per unit span $m_{00} = 1.997$ and 0.4992 kg/m, moments of inertia per unit span $m_{22} = m_{33} = 42.60$ and 2.662 mg·m^2/m for Bars 1 and 2, and Bar 3, respectively.

Table 1: Sectional stiffness properties of the bars

	Axial S [MN]	Shearing K_{22} [MN]	Shearing K_{33} [MN]	Torsional H_{11} [N·m^2]	Bending H_{22} [N·m^2]	Bending H_{33} [N·m^2]
Bar 1 & 2	52.99	16.88	16.88	733.5	1131	1131
Bar 3	13.25	4.220	4.220	45.84	70.66	70.66

The bars four corners are labeled points $\mathbf{C_1}$, $\mathbf{C_2}$, $\mathbf{C_3}$, and $\mathbf{C_4}$; the mid-points of the edges are labeled points $\mathbf{M_1}$, $\mathbf{M_2}$, $\mathbf{M_3}$, and $\mathbf{M_4}$, as illustrated in fig 2. If the bars were infinitely rigid, no motion would be possible because the mechanism locks. For elastic bars, motion becomes possible, but generates large, rapidly varying internal forces and moments.

This problem was simulated for a total of 12 s using 3000 time steps of constant size $\Delta t = 4$ ms. If the four revolute joints had their axes of rotation orthogonal to the plane of the mechanism, the response of the system would be purely planar, and bars 1 and 3 would rotate at constant angular velocities around points **A** and **D**, respectively. The initial defect in the mechanism causes a markedly different response. Bar 1 rotates at the constant prescribed angular velocity, but bar 3 now oscillates back and forth, never completing an entire turn.