Lateral buckling of a thin beam

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If a beam is bent in its plane of greatest flexural rigidity, lateral buckling will occur when a critical value of the transverse load is reached. In this benchmark problem, the tip of a beam is subjected to a transverse load applied through a crank and link mechanism, as depicted in fig. 1. The beam is clamped at one end, while the other end is connected to the link through a spherical joint. The crank and link are modeled by flexible beams connected by revolute joints. As the crank rotates, the beam tip is pushed up. When the buckling load is reached, the beam snaps laterally and becomes significantly softer in bending due to the pronounced twisting deformation.

Figure 1 depicts the configuration of the problem. The beam is of length $L = 1$ m, the crank and link lengths are $L_c = 0.05$ m and $L_\ell = 0.25$ m, respectively. The rotation of the crank is prescribed a $\phi = \pi(1 - \cos \pi t/T)/2$, for $t \leq T$ and $\phi = \pi$ for $t > T$, where $T = 0.4$ s. To simulate an initial imperfection of the system, the tip of the beam is connected to the spherical joint via a rigid body connection of length $d = 0.1$ mm. The plane of the crank and link mechanism is offset from the plane of the beam by the same distance d.

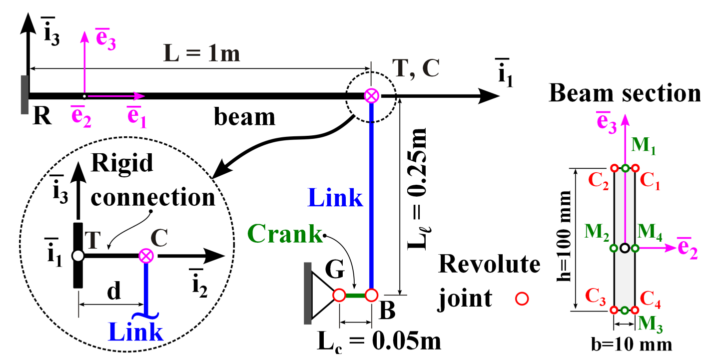

Figure 1: Configuration of the crank actuated beam.

The beam features a rectangular cross-section of width $b = 10$ mm and height $h = 100$ mm. The beam's four corners are labeled points $\mathbf{C_1}$, $\mathbf{C_2}$, $\mathbf{C_3}$, and $\mathbf{C_4}$; the mid-points of the edges are labeled points $\mathbf{M_1}$, $\mathbf{M_2}$, $\mathbf{M_3}$, and $\mathbf{M_4}$, as illustrated in fig. 1. The link has a circular cross-section of radius $R_\ell = 12$ mm. Finally, the crank also features circular cross-section, but its radius is $R_c = 24$ mm. All components are made of aluminum, whose mechanical characteristics are Young's modulus $E = 73$ GPa and Poisson's ratio $\nu = 0.3$. These physical properties translate to the sectional stiffness properties listed in table 1. The sectional mass properties are as follows: mass per unit span $m_{00} = 2.68$, 1.212, and 4.85 kg/m, moments of inertia per unit span $m_{22} = 2233$, 43.65, and 698.3, $m_{33} = 22.33$, 43.65, and 698.3 mg·m^2/m for the beam, link, and crank, respectively.

Table 1: Sectional stiffness properties of the beams

	Axial	**Shearing**	**Shearing**	**Torsional**	**Bending**	**Bending**
	S [MN]	K_{22} [MN]	K_{33} [MN]	H_{11} [N·m^2]	H_{22} [N·m^2]	H_{33} [N·m^2]
Beam	73	5.025	23.40	877.2	60830	608.3
Link	33.02	10.81	10.81	914.5	1189	1189
Crank	132.1	43.22	43.22	14630	19020	19020