The rotating shaft

Olivier A. Bauchau
University of Michigan-Shanghai Jiao Tong University Joint Institute,
Shanghai, China

1 Description

A flexible shaft of length L is supported at its ends by bearings, as depicted in fig. 1. At point **R**, the shaft is connected to the ground by means of a revolute joint and the angular speed of the shaft is a prescribed function of time, $\Omega(t)$. At point **T**, the shaft is connected to the ground via a cylindrical joint. A rigid disk is attached to the shaft at its mid-span point **M**. The disk's center of mass is located a distance d above the reference axis of the shaft, thereby creating an initial unbalance of the system.

At the initial time, the shaft is at rest and deformed under the effect of the gravity loads acting in the vertical direction indicated in the figure. The shaft is set in motion by prescribing its rotation at point **R** and lateral oscillations ensue due to the initial imperfection of the system. As the shaft accelerates, it passes through the first natural bending frequency of the system and the operation goes from sub- to super-critical. As predicted by linear rotor dynamics theory, the shaft becomes unstable when operating at the critical speed. In the present example, the magnitudes of lateral oscillations and corresponding internal forces rise as the shaft is accelerated through the critical speed.

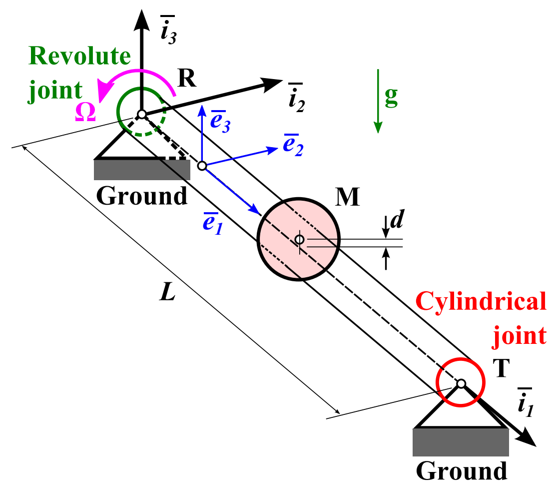

Figure 1: Configuration of the rotating shaft

The shaft has a length $L = 6$ m and is made of steel (density $\rho = 7800$ kg/m^3, Young's modulus $E = 210$ GPa, and Poisson's ratio $\nu = 0.3$). The cross-section is annular with inner and outer radii $r_I = 0.045$ and $r_O = 0.05$ m, respectively. The shaft's sectional stiffness properties are summarized in Table 1. The mass per unit span is $m = 11.64$ kg/m, the moments of inertia per unit span are $m_{22} = m_{33} = 13.17$ g·m^2/m and the resulting polar moment of inertia per unit span is $m_{11} = m_{22} + m_{33} = 26.34$ g·m^2/m.

The mid-span circular disk is of mass $m_d = 70.573$ kg, radius $r_d = 0.24$ m, and thickness $t_d = 0.05$ m. Its inertial tensor computed with respect to the center of mass is diagonal, diag(2.0325, 1.0163, 1.0163) g·m^2. Its center of mass is located a distance $d = 0.05$ m above the shaft reference axis. The acceleration of gravity is $g = 9.81$ m/s^2.

Axial S [MN]	Shearing K_{22} [MN]	Shearing K_{33} [MN]	Torsional H_{11} [kN·m^2]	Bending H_{22} [kN·m^2]	Bending H_{33} [kN·m^2]
313.4	60.5	60.5	272.7	354.5	354.5

Table 1: Sectional stiffness properties of the shaft

The shaft's rotation at point **R** is prescribed to be

$$\Omega(t) = \begin{cases} A_1\omega[1-\cos(\pi t/T_1)]/2, & 0 \le t \le T_1, \\ A_1\omega, & T_1 < t \le T_2, \\ A_1\omega + (A_2 - A_1)\omega\{1-\cos[\pi(t-T_2)/(T_3-T_2)]\}/2, & T_2 < t \le T_3, \\ A_2\omega, & t > T_3, \end{cases}$$

where $A_1 = 0.8$, $A_2 = 1.2$, $T_1 = 0.5$ s, $T_2 = 1$ s, $T_3 = 1.25$ s and $\omega = 60$ rad/s is close to the first natural frequency of the shaft in bending ($\omega_1 = 56.7$ rad/s).