

# *Dymore User's Manual*

## Definition and formulation of hydraulic devices

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## 1 Introduction

The behavior of hydraulic actuators and dampers can be modeled in several manners. In the first approach, very simple idealizations of hydraulic components are used. For instance, a hydraulic damper would be idealized as a dashpot: the force in the damper is proportional to the relative velocity of the piston. More often than not, actual damper will exhibit a nonlinear force-velocity relationship and a linear approximation is clearly too crude. The accuracy of the predictions might be improved if the damper is modeled as a nonlinear dashpot; this approach is widely used in industry. In that case, the nonlinear characteristics of the device are identified by a number of bench experiments, typically involving harmonic excitations of the device at various frequencies and amplitudes. The main drawback of this approach is that physical characteristics identified under harmonic excitation might not yield good results when the device is subjected to arbitrary excitation in time.

In the second approach, the hydrodynamic behavior of the device is linearized to obtain one or more ordinary differential equations relating control inputs to the forces generated by the device; typical equations are given in text books such as those of Viersma [1] or Canon [2]. While this approach is physics based and captures some basic aspects of hydraulic devices, the linearization process is clearly too restrictive. In fact, rotorcraft lead-lag dampers are often purposely designed to behave in a nonlinear manner. Indeed, a linear device would generate high damping forces under high stroking rates; these high forces must be reacted at the hub and at the root of the blade, creating high stresses and decreasing fatigue life. A possible remedy to this situation is to use pressure relief valves that act as force limiters, implying a nonlinearity essential to the design and behavior of the device.

In the last approach, a physics based, fully nonlinear representation of hydraulic devices is implemented. This enables the determination of the complex interaction phenomena between the structural and actuator dynamics: pressure levels in the hydraulic chambers are now coupled with the dynamic response of the system. This paper describes such an approach in detail, and its predictions are validated against bench test measurements and flight test data for Sikorsky's UH-60 aircraft.

The modeling of hydraulic devices has been the subject of detailed studies. Welsh [3] proposed a detailed model for predicting the dynamic response of helicopter air-oil landing gear that included several degrees of freedom representing the tire, floating piston, orifice piston, and simple fluid and adiabatic gas models. In

a later effort [4], the same author addressed the problem of modeling the lubrication system of a helicopter using a similar approach. In both cases, detailed models of hydraulic systems were developed, but not coupled with the dynamic response of the vehicle.

A variety of hydraulic devices are used in the rotorcraft industry: hydraulic actuators are crucial components of many main rotor control systems, hydraulic lead-lag dampers are used in many rotor designs, and landing gear often involve hydraulic or pneumatic elements. In the case of lead-lag dampers, the hydraulic device tightly interacts with rotor response; in fact, blade root edgewise moments depend to a large extent on damper response characteristics. To deal with this variety of devices, a modular approach is taken. At first, models are developed for three basic hydraulic elements: the hydraulic chamber, the hydraulic orifice, and the pressure relief valve. Models for entire hydraulic devices are then constructed by assembling the models of a number these hydraulic elements. In this work, models for hydraulic actuators, simple hydraulic dampers, and hydraulic dampers with pressure relief valves are discussed.

Once a model of the hydraulic device is in hand, it is to be coupled with a comprehensive rotorcraft simulation code. In this effort, hydraulic device models are coupled to a finite element based multibody formulation of a helicopter rotor system within a comprehensive analysis [5]. Within the framework of flexible mechanism analysis codes, the modeling of hydraulic devices has attracted limited attention; Cardona and Géradin [6] proposed models for a hydraulic jack and for the actuator of an aircraft retractable landing gear.

## 2 Basic hydraulic elements

Hydraulic devices can be seen as an assembly of simple hydraulic elements; in this work, three basic hydraulic elements will be presented: hydraulic chambers, orifices, and pressure relief valves. These basic elements are described in the following sections. Of course, a variety of other elements could be developed, such as hydraulic accumulators or check valves.

### 2.1 Hydraulic chamber

The hydraulic chamber, shown in fig. 1, is probably the most common hydraulic component. The chamber, of volume  $V$  and cross-sectional area  $A$ , is filled with a hydraulic fluid of bulk modulus  $B$  under pressure  $p$ . Often, due to the presence of a piston, the length of the chamber can vary. The change in length of the chamber, due to piston motion, is denoted  $d$ . Finally, hydraulic fluid can flow into the chamber;  $Q$  denotes the net volumetric flow rate into the chamber.

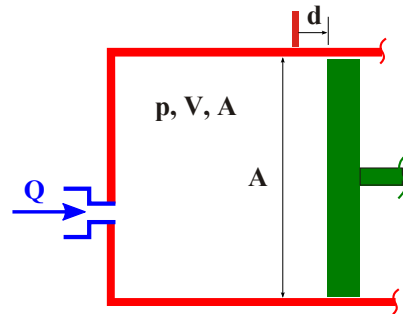


Figure 1: Configuration of a hydraulic chamber.

The evolution of the pressure in the chamber is governed by the following first order differential equation

$$\dot{p} = \frac{B}{V} (Q - A\lambda\dot{d}). \quad (1)$$

The factor  $\lambda$  is a configuration dependent parameter: if a positive value of  $d$  increases the volume of the chamber,  $\lambda = +1$ ;  $\lambda = -1$  in the opposite case. The instantaneous volume of the chamber is

$$V = V_0 + A\lambda d, \quad (2)$$

where  $V_0$  is the initial volume of the hydraulic chamber. The bulk modulus of the hydraulic fluid is a function of the fluid pressure; an accurate approximation of this dependency is written as

$$B = \frac{1 + \alpha p + \beta p^2}{\alpha + 2\beta p}, \quad (3)$$

where  $\alpha$  and  $\beta$  are physical constants for the hydraulic fluid [1].

## 2.2 Hydraulic orifice

The hydraulic orifice, shown in fig. 2, allows the flow of hydraulic fluid through an orifice of sectional area  $A_{\text{orf}}$ . The orifice is connected to two hydraulic chambers with pressures  $p_0$  and  $p_1$ , respectively. A pressure differential,  $\Delta p = p_0 - p_1$ , will drive a volumetric flow rate,  $Q_{\text{orf}}$ , across the orifice; the positive direction of this flow rate is indicated on the figure. The magnitude of this volumetric flow rate is related to the pressure differential by the following equation

$$Q_{\text{orf}} = A_{\text{orf}} C_d \sqrt{\frac{2|\Delta p|}{\rho}} \frac{\Delta p}{|\Delta p|}, \quad (4)$$

where  $\rho$  denotes the mass density of the hydraulic fluid. For turbulent flow conditions, the theoretical value of the discharge coefficient, denoted  $C_d$ , is  $C_d = 0.611$ , see Viersma [1].

## 2.3 Pressure relief valve

The pressure relief valve, shown in fig. 3, is connected to two hydraulic chambers with pressures  $p_0$  and  $p_1$ , respectively. It features a poppet of mass  $m$  connected to a spring of stiffness constant  $k$  and dashpot of viscous constant  $c$ ; the spring is preloaded with a pre-load force. The equation of motion of the pressure relief valve is:

$$m \ddot{x} + c \dot{x} + kx = (p_0 A_0 - p_1 A_1) - F_p, \quad (5)$$

where  $F_p$  is the pressure relief valve pre-load force. When the net force acting on the poppet is smaller than the pre-load force, *i.e.* when  $p_0 A_0 - p_1 A_1 < F_p$ , the valve remains closed,  $x = \dot{x} = 0$ . On the other hand, when the net force is large enough to overcome the pre-load force, the poppet opens and its motion is governed by eq. (5). Once the valve is open, fluid will flow through the valve at the following volumetric rate

$$Q_{\text{prvl}} = A_{\text{prvl}} C_d \sqrt{\frac{2|\Delta p|}{\rho}} \frac{\Delta p}{|\Delta p|}, \quad (6)$$

where  $\Delta p = p_0 - p_1$  is the pressure differential across the valve. The area  $A_{\text{prvl}}$  through which the fluid flows is a function of the valve opening

$$A_{\text{prvl}} = \begin{cases} ax + bx^2, & x > 0, \\ 0 & x \leq 0, \end{cases} \quad (7)$$

where  $a$  and  $b$  are coefficients defining the pressure relief valve sectional area.

The pressure relief valve acts as a pressure regulator: when the pressure differential across the valve becomes high enough,  $p_0 A_0 > p_1 A_1 + F_p$ , the valve opens and the ensuing flow tends to equilibrate the pressures, at which point, the valve closes.

## 3 Hydraulic devices

The hydraulic elements described in the previous section can be combined to form practical hydraulic devices such as linear hydraulic actuators, simple dampers, or dampers with pressure relief valves that are described below. More complex devices could be modeled using the same technique.

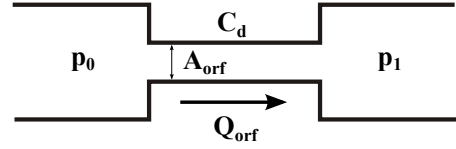


Figure 2: Configuration of a hydraulic orifice.

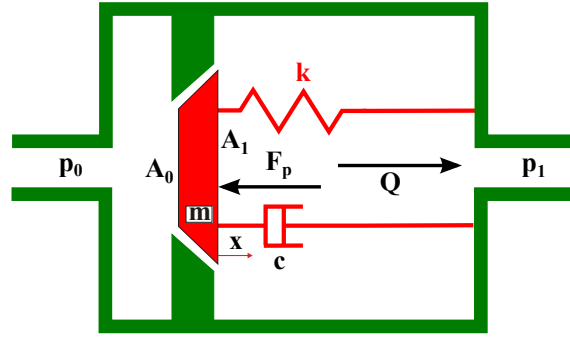


Figure 3: Configuration of a pressure relief valve.

### 3.1 Hydraulic linear actuator

The linear hydraulic actuator combines two hydraulic chambers, chamber 0 and chamber 1, and two orifices, orifice 0 and orifice 1, to form the configuration depicted in fig. 4. The hydraulic chamber 0 and chamber 1 are under pressures  $p_0$  and  $p_1$ , respectively; note that the  $\lambda$  factors are  $+1$  and  $-1$  for the two chambers, respectively. The hydraulic orifice 0 and orifice 1 generate flow rates  $Q_0$  and  $Q_1$  into chambers 0 and 1, respectively. The two orifices have entrance pressures  $p_{E0}$  and  $p_{E1}$ , respectively. To increase the length of the actuator, control valves (not part of the present model) will set the entrance pressure of orifice 0 to a high value,  $p_h$ , such that  $p_{E0} = p_h$ , while the entrance pressure of orifice 1 remains at a low value,  $p_s$ , such that  $p_{E1} = p_s$ ;  $p_s$  denotes the hydraulic circuit background pressure. To decrease the length of the actuator, the control valves reverse the pressure level at the entrance to the two orifices.

The force generated by the actuator, denoted  $F^h$ , is

$$F^h = p_0 A_0 - p_1 A_1. \quad (8)$$

The governing equations for the linear hydraulic actuator include equations for the pressures  $p_0$  and  $p_1$  in the two chambers, eq. (1), and equations for the flow rates  $Q_0$  and  $Q_1$  through the orifices, eq. (4).

Most hydraulic actuators are also equipped with check valves that connect the hydraulic chambers to the circuit background pressure when the chamber pressure falls below the background pressure, in an effort to avoid cavitation in the chamber.

### 3.2 Simple hydraulic damper

The simple hydraulic damper combines two hydraulic chambers, chamber 0 and chamber 1, and one orifice connecting the two chambers to form the configuration depicted in fig. 5. The hydraulic chamber 0 and chamber 1 are under pressures  $p_0$  and  $p_1$ , respectively; note that the  $\lambda$  factors are  $+1$  and  $-1$  for the two chambers, respectively. The hydraulic orifice generates a flow rate  $Q$  from chamber 0 into chamber 1. If the length of the damper increases (*i.e.* piston and rod move to the right in fig. 5, pressure  $p_1$  increases whereas pressure  $p_0$  decreases. This generates a pressure differential across the orifice and hence, a flow rate  $Q$  into chamber 0 that tends to equilibrate the pressures in the chambers. The force generated by the damper always opposes the motion and is therefore a damping force.

The force generated by the damper is here again given by eq. (8). The governing equations for the simple hydraulic damper include equations for the pressure  $p_0$  and  $p_1$  in the two chambers, eq. (1), and one equation for the flow rate  $Q$  through the orifice, eq. (4).

### 3.3 Hydraulic damper with pressure relief valves

Rotorcraft hydraulic lead-lag dampers present a configuration similar to that of the simple damper described in the previous section. However, this simple design suffers an important drawback: under a high stroking

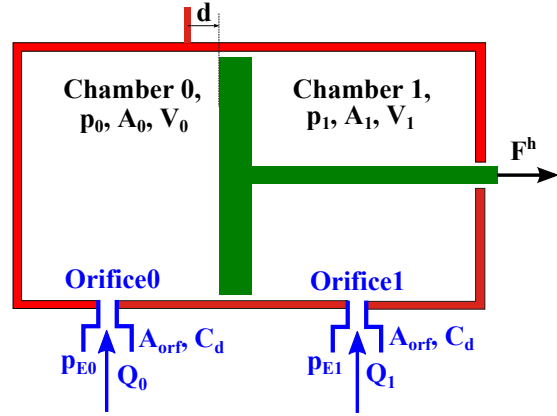


Figure 4: Configuration of the hydraulic actuator.

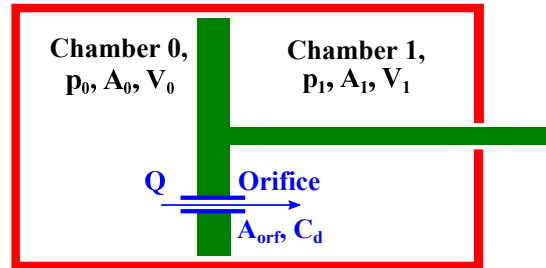


Figure 5: Configuration of the simple hydraulic damper.

rate, the pressure differential in the chambers can be rather high, and hence, high damping forces are generated. These high forces must be reacted at the hub and at the root of the blade, creating high stresses and decreasing fatigue life. To limit the forces in the hydraulic damper, two pressure relief valves are added to the configuration, as shown in fig. 6. The new design combines two hydraulic chambers, chamber 0 and chamber 1, one orifice connecting the two chambers and two pressure relief valves, valve 0 and valve 1. The hydraulic chamber 0 and chamber 1 are under pressures  $p_0$  and  $p_1$ , respectively; note that the  $\lambda$  factors are  $+1$  and  $-1$  for the two chambers, respectively. The hydraulic orifice generates a flow rate  $Q$  from chamber 0 into chamber 1. Finally, when open, the pressure relief valves regulate the pressures in chambers 0 and 1.

If the length of the damper increases, pressure  $p_1$  increases whereas pressure  $p_0$  decreases. This generates a pressure differential across the orifice and hence, a flow rate  $Q$  into chamber 0 that tends to equilibrate the pressures in the chambers. If the stroking rate is high, the pressure differential in the chambers will become high enough to open pressure relief valve 1, resulting in an additional flow rate  $Q_1$  from chamber 1 into chamber 0. Given the sign of the pressure differential, valve 0 will remain closed. The opening of the valve and the ensuing flow controls the magnitude of the pressure differential, and hence of the damper force that is still given by eq. (8). The force generated by the damper always opposes the motion and is therefore a damping force. In practical designs, hydraulic dampers are also equipped with check valves, as discussed for actuators. The hydraulic chambers are connected to a plenum with oil at the circuit background pressure to prevent pressure drops in the chambers and subsequent cavitation.

The governing equations for the hydraulic damper with pressure relief valves include equations for the pressures  $p_0$  and  $p_1$  in the two chambers, eq. (1), one equation for the flow rate  $Q$  through the orifice, eq. (4), two equations of motion for the valve poppets, eq. (5), and two equations for the flow rates through the valves, eq. (6). The flow area of the valves is computed with the help of eq. (7).

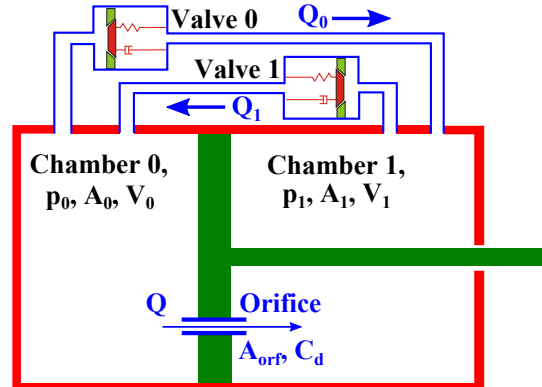


Figure 6: Configuration of the hydraulic damper with pressure relief valves.

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