

# *Dymore User's Manual*

## Formulation of spring elements

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## 1 Representation of spring characteristics

Spring characteristics are defined by the coefficients of the expansion of the force-stretch relationship in term of Chebyshev polynomials

$$F^e(\mu) = \sum_{i=1}^N c_i T_{i-1}(\mu), \quad (1)$$

where  $T_i$  are the Chebyshev polynomials,  $c_i$  the coefficients of the expansion,  $N$  the number of terms in the expansion, and  $\mu$  the non-dimensional stretch defined as

$$\mu = \frac{2s - (x_{hi} + x_{lo})}{x_{hi} - x_{lo}}. \quad (2)$$

$x_{lo}$  and  $x_{hi}$  are the lower and upper bounds defining the range over which the approximation is valid. If the stretch of the spring goes beyond this range during the simulation an error message will be printed.

## 2 Formulation of spring elements

The spring element generates an elastic force,  $F^e$ , that can be derived from a potential,  $V(s)$ , such that

$$F^e = \frac{dV(s)}{ds}. \quad (3)$$

The potential of the elastic forces is a *positive definite function of the stretch*, *i.e.*,  $V(s) \geq 0, \forall s$ . In the present implementation, the elastic force, rather than the potential, is given as a function of the spring stretch,  $s$ . The stiffness of the

spring, denoted  $k$ , is then obtained by taking the derivative of the elastic force with respect to the stretch,

$$k = \frac{dF^e}{ds} = \frac{d^2V(s)}{d^2s}. \quad (4)$$