Dymore User's Manual

Computing an orientation from the twist angle

In general, triads with an arbitrary orientation can be defined, and three parameters are required to unequivocally define the triad. When dealing with beams, however, the triads must be such that vector $\bar{e}_1$ is tangent to the curve defining the beam, whereas vectors $\bar{e}_2$ and $\bar{e}_3$ define the plane of the cross-section, which is normal to the curve, as depicted in fig. 1. Since vector $\bar{e}_1$ is known, a single parameter only is required to fully define the orientation of the triad. A convenient way of defining the orientation of the triad is then to define the twist angle ϕ, *measured in degrees*, shown in fig. 1. The corresponding triad at a point is then constructed as follows.

1. Compute the unit tangent vector to the curve to find $\bar{e}_1^T = \lfloor e_{11}, e_{12}, e_{13} \rfloor$.
2. Next, compute vector $\bar{e}_2$ based on the twist angle.

$$\bar{e}_2 = \frac{1}{\Delta} \left\{ \begin{array}{c} -\dfrac{e_{12}\cos\phi + e_{13}\sin\phi}{e_{11}} \\ \cos\phi \\ \sin\phi \end{array} \right\}; \quad \Delta = \sqrt{1 + \left(\frac{e_{12}\cos\phi + e_{13}\sin\phi}{e_{11}} \right)^2}. \tag{1}$$

   It is readily verified that $\|\bar{e}_2\| = 1$, *i.e.*, $\bar{e}_2$ is a unit vector; $\bar{e}_1^T \bar{e}_2 = 0$, *i.e.*, $\bar{e}_2$ is normal to $\bar{e}_1$; and finally, the projection of vector $\bar{e}_2$ onto plane $(\bar{b}_2, \bar{b}_3)$, denoted $\bar{e}_2'$, makes an angle ϕ with axis $\bar{b}_2$. Indeed, $\bar{e}_2'^T = \lfloor 0, \cos\phi, \sin\phi \rfloor / \Delta$, and hence $(\bar{b}_2^T \bar{e}_2')/\|\bar{e}_2'\| = (\Delta \cos\phi)/\Delta = \cos\phi$. Clearly, the use of the twist angle to define triads requires that $e_{11} \neq 0$.

3. Finally, compute vector $\bar{e}_3 = \widetilde{e}_1 \bar{e}_2$.

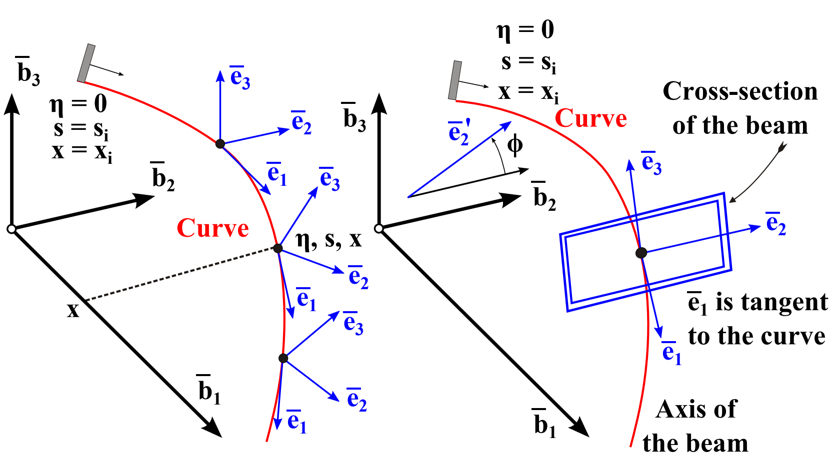

Figure 1: Left figure: A field of orientations along a curve. Right figure: Using a twist angle to define the orientation of the cross-section of a beam.