Dymore User's Manual
The Coulomb and Lugre friction models

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1 The Coulomb model

Coulomb's classical law for dry friction is used to model dry friction. It states that the friction force, $\underline{F}^f$, is proportional to the magnitude of the normal contact force $|F^n|$

$$\underline{F}^f = -\mu_k \ |F^n| \ \frac{\underline{V}_r}{V_r}, \tag{1}$$

where μ_k is the *coefficient of dynamic friction* and $V_r = \|\underline{V}_r\|$ the magnitude of the relative velocity in the plane tangent to the bodies at the contact point. If the relative velocity vanishes, sticking (or rolling) takes place. In this case, the frictional force is

$$\|\underline{F}^f\| \leq \mu_s \ |F^n|, \tag{2}$$

where μ_s is the *coefficient of static friction*. Sliding gives way to sticking when the relative velocity vanishes, *i.e.* when $V_r = 0$.

To successfully treat sticking, sliding and transition regimes, it is necessary to properly model the frictional forces. Application of Coulomb's law involves discrete transitions from sticking to sliding and vice-versa, as dictated by the vanishing of the relative velocity and the magnitude of the friction force. These discrete transitions can cause numerical difficulties, and the use of a *continuous friction law* becomes necessary. Typically, the friction coefficient is regularized to obtain a continuous friction law written as

$$\underline{F}^f = -|F^n| \ \frac{\underline{V}_r}{V_r} \ \mu_k \tanh(V_r/v_s), \tag{3}$$

where v_s is a characteristic velocity usually chosen to be small compared to the maximum relative velocity encountered during the simulation. The dynamic coefficient of friction has been regularized as $\mu_k \tanh(V_r/v_s)$ to smooth out the friction force discontinuity, as depicted in fig. 1. The continuous friction law describes both sliding and sticking behavior; sticking (or rolling) is replaced by "creeping" of the body with respect to the plane with a small relative velocity.

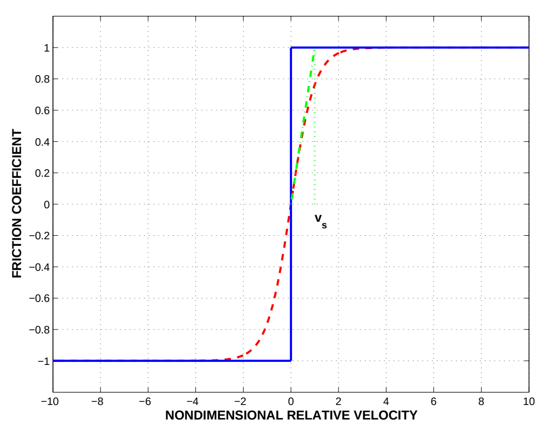

Figure 1: The friction coefficient for Coulomb's friction law (solid line), and the continuous friction law (dashed line). $\mu_k = 1.0$.

When the Coulomb model is used, it is necessary to switch on the *time adaptivity* feature for the time stepping procedure. When friction occurs, the friction model will dictate the time step for the analysis, as determined by the **strategy parameters**. In view of the rapid variation of the continuous friction law for small relative velocities, the time step size must be reduced when the relative velocity is of the order of v_s. To achieve this goal, the time step size for the next time step is selected as

$$\Delta t_{\text{new}} = \Delta t_{\text{min}} + (\Delta t_{\text{max}} - \Delta t_{\text{min}}) \tanh^4 \nu, \tag{4}$$

where ν, defined as

$$\nu = \frac{V_r/v_s}{\bar{v}_{\text{min}}}, \tag{5}$$

measures the smallness of the relative velocity. The *default values* of the strategy parameters are $c_1 = \Delta t_{\text{min}} = 1.0 \ e^{-04}$ sec, $c_2 = \Delta t_{\text{min}} = 1.0 \ e^{-03}$ sec, $c_3 = \bar{v}_{\text{min}} = 5$ and $c_4 = T_s = 1.0$ sec.

2 The LuGre model

The LuGre model [1] is an advanced model for friction. This model uses the following equation for the friction coefficient

$$\mu = \sigma_0 z + \sigma_1 \frac{\mathrm{d}z}{\mathrm{d}t} + \sigma_2 V_r, \tag{6}$$

where μ is the instantaneous friction coefficient and V_r the relative velocity; σ_0, σ_1, and σ_2 are coefficients of the LuGre model. The variable z is an internal state of the LuGre model and its evolution is governed by the following differential equation

$$\frac{\mathrm{d}z}{\mathrm{d}t} = V_r - \frac{\sigma_0 |V_r|}{g(V_r)}\, z; \quad g(V_r) = \mu_k + (\mu_s - \mu_k) \exp(-|V_r/v_s|^\gamma), \tag{7}$$

where γ is an additional parameter of the LuGre model. Note that if $\sigma_1 = \sigma_2 = 0.0$ and $\mu_s = \mu_k$, the LuGre model becomes identical to Dahl's model [2].

When the LuGre model is used, it is necessary to switch on the *time adaptivity* feature for the time stepping procedure. When friction occurs, the friction model will dictate the time step for the analysis, as determined by the **strategy parameters**. In view of the rapid variation of the function $g(V_r)$ for small relative velocities, the time step size must be reduced when the relative velocity is of the order of v_s. To achieve this goal, the time step size for the next time step is selected as

$$\Omega \Delta t_{\mathrm{new}} = \Delta\tau_{\min} \begin{cases} 1 & \nu \leq 1 \\ \nu^\alpha & \nu > 1 \end{cases}, \tag{8}$$

where $\Omega = \sigma_0 v_s/\mu_k$ is the time constant of the LuGre model and the quantity ν, defined as

$$\nu = \frac{V_r/v_s}{\bar{v}_{\min}}, \tag{9}$$

measures the smallness of the relative velocity. The *default values* of the strategy parameters are $c_1 = \Delta\tau_{\min} = 0.02$, $c_2 = \bar{v}_{\min} = 5$, $c_3 = \alpha = 1.2$and $c_4 = \bar{T}_s = 5.0$.

References

[1] C. Canudas de Wit, H. Olsson, K.J. Astrom, and P. Lischinsky. A new model for control of systems with friction. *IEEE Transactions on Automatic Control*, 40:419–425, 1995.

[2] P.R. Dahl. Solid friction damping of mechanical vibrations. *AIAA Journal*, 14:1675–1682, 1976.