Dymore User's Manual

Definition of membrane properties

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1 Membrane property definition

The physical mass and stiffness properties of a membrane are defined in this section. In general, the stiffness characteristic of a membrane can be represented in the following matrix form

$$\underline{N} = \underline{\underline{A}}\,\underline{\varepsilon},$$

where $\underline{N}^T = \{N_{11}, N_{22}, N_{12}\}$ is the vector of in-plane forces per unit length of the membrane and $\underline{\varepsilon}^T = \{\epsilon_{11}, \epsilon_{22}, \epsilon_{12}\}$ the corresponding in-plane strains. All loading and strain quantities are measured in a material frame of reference. The thickness of the membrane is allowed to vary over the surface that defined the membrane. Hence, the physical properties will be defined in a non-dimensional manner with respect to thickness.

If the membrane is made of an *isotropic*, linearly elastic material, the in-plane stiffness matrix is defined as

$$\underline{\underline{\bar{A}}} = \frac{1}{h}\underline{\underline{A}} = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \dfrac{1-\nu}{2} \end{bmatrix},$$

where h is the membrane thickness, E Young's modulus, and ν Poisson's ratio.

If the membrane is made of an *anisotropic*, linearly elastic material, the in-plane stiffness matrix is defined as

$$\underline{\underline{\bar{A}}} = \frac{1}{h}\underline{\underline{A}} = \begin{bmatrix} \bar{a}_1 & \bar{a}_4 & \bar{a}_7 \\ \bar{a}_2 & \bar{a}_5 & \bar{a}_8 \\ \bar{a}_3 & \bar{a}_6 & \bar{a}_9 \end{bmatrix}.$$

Damping in the membrane can be modeled by viscous forces $\underline{N}_d^*$ proportional to the strain rates, $\underline{N}_d^* = \mu_s \underline{\underline{A}}\,\dot{\underline{\varepsilon}}^*$, where μ_s is the damping coefficient.