# *Dymore User's Manual*

## Definition of shell properties

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# 1 Shell stiffness properties

The physical mass and stiffness sectional properties of a shell are defined in this section. In general, the stiffness characteristic of a shell can be represented in the following matrix form

$$\underline{F} = \begin{Bmatrix} \underline{N} \\ \underline{T} \\ \underline{M} \end{Bmatrix} = \begin{bmatrix} \underline{\underline{A}} & \underline{\underline{F}} & \underline{\underline{B}} \\ \underline{\underline{F}}^T & \underline{\underline{S}} & \underline{\underline{G}} \\ \underline{\underline{B}}^T & \underline{\underline{G}}^T & \underline{\underline{D}} \end{bmatrix} \begin{Bmatrix} \underline{\varepsilon} \\ \underline{\gamma} \\ \underline{\kappa} \end{Bmatrix} = \underline{\underline{C}}^* \begin{Bmatrix} \underline{\varepsilon} \\ \underline{\gamma} \\ \underline{\kappa} \end{Bmatrix} = \underline{e}, \tag{1}$$

where $\underline{N}^T = \{N_{11}, N_{22}, N_{12}\}$ is the vector of in-plane forces per unit length of the shell and $\underline{\varepsilon}^T = \{\epsilon_{11}, \epsilon_{22}, \epsilon_{12}\}$ the corresponding in-plane strains; $\underline{T}^T = \{T_{13}, T_{23}\}$ is the vector of transverse shearing forces per unit length of the shell and $\underline{\gamma}^T = \{\gamma_{13}, \gamma_{23}\}$ the corresponding transverse shearing strains; $\underline{M}^T = \{M_{11}, M_{22}, M_{12}\}$ is the vector of bending moments per unit length of the shell and $\underline{\varepsilon}^T = \{\kappa_{11}, \kappa_{22}, \kappa_{12}\}$ the corresponding curvatures. All loading and strain quantities are measured in a material frame of reference. The sign conventions for these various load components are depicted in fig. 1.

The thickness of the shell is allowed to vary over the surface that defined the shell. Hence, the physical properties will be defined in a non-dimensional manner with respect to thickness.

If the shell is assumed to be made of a homogeneous, isotropic, linearly elastic material, Young's modulus, $E$, and Poisson's ratio, $\nu$, then completely define the material stiffness characteristics. The in-plane stiffness matrix is

$$\bar{\underline{\underline{A}}} = \frac{1}{h}\underline{\underline{A}} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \dfrac{1-\nu}{2} \end{bmatrix}, \tag{2}$$

where $h$ is the shell thickness. The bending stiffness matrix is

$$\bar{\underline{\underline{D}}} = \bar{\underline{\underline{A}}}; \quad D = \frac{h^3}{12}\,\bar{\underline{\underline{D}}}. \tag{3}$$

Finally, the shearing stiffness matrix is

$$\bar{\underline{\underline{S}}} = \frac{1}{h}\underline{\underline{S}} = \frac{5}{6}\,\frac{E}{2(1+\nu)} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \tag{4}$$

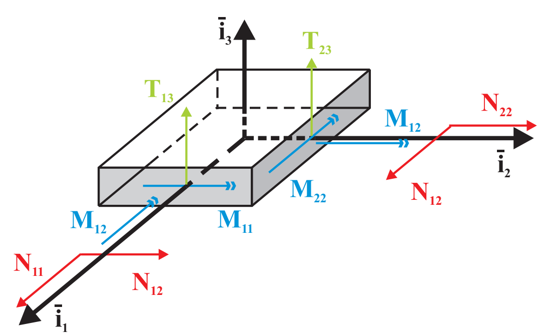


Figure 1: Sign convention for the loads acting on a shell element.

The other stiffness matrices all vanish.

If the keyword @STIFFNESS_MATRIX_A appears first, three stiffness matrices must be defined. Shells made of advanced composite materials with arbitrary lay-ups can be defined in this manner. The coupling stiffness matrices $B$, $F$, and $G$ are assumed to vanish unless defined below. The in-plane stiffness matrix, $\underline{\underline{A}}$, is defined as

$$\bar{\underline{\underline{A}}} = \frac{1}{h}\underline{\underline{A}} = \begin{bmatrix} \bar{a}_1 & \bar{a}_4 & \bar{a}_7 \\ \bar{a}_2 & \bar{a}_5 & \bar{a}_8 \\ \bar{a}_3 & \bar{a}_6 & \bar{a}_9 \end{bmatrix}. \tag{5}$$

The bending stiffness matrix, $\underline{\underline{D}}$, is defined as

$$\underline{\underline{\bar{D}}} = \frac{12}{h^3}\underline{\underline{D}} = \begin{bmatrix} \bar{d}_1 & \bar{d}_4 & \bar{d}_7 \\ \bar{d}_2 & \bar{d}_5 & \bar{d}_8 \\ \bar{d}_3 & \bar{d}_6 & \bar{d}_9 \end{bmatrix}. \tag{6}$$

The shearing matrix $\underline{\underline{S}}$ is defined as

$$\underline{\underline{\bar{S}}} = \frac{1}{h}\underline{\underline{S}} = \begin{bmatrix} \bar{s}_1 & \bar{s}_3 \\ \bar{s}_2 & \bar{s}_4 \end{bmatrix}. \tag{7}$$

Note that these three matrices have the same units as a stiffness modulus.

When the shell is made of anisotropic material, elastic coupling can occur between the in-plane and bending behaviors (stiffness matrix $\underline{\underline{B}}$), in-plane and shearing behaviors (stiffness matrix $\underline{\underline{F}}$), or shearing and bending behaviors (stiffness matrix $\underline{\underline{G}}$). Any of these three matrices can be optionally defined.

Damping in the shell can be modeled by viscous forces $\underline{F}^*_d$ proportional to the strain rates, $\underline{F}^*_d = \mu_s\ C^*\ \underline{\dot{e}}^*$, where $\mu_s$ is the damping coefficient.

# 2 Shell mass properties

The mass properties of the shell are defined by three coefficients corresponding to integrals of the mass distribution through the thickness of the shell

$$m = \int_h \rho\ \mathrm{d}\zeta; \quad m^* = \int_h \rho\zeta\ \mathrm{d}\zeta = m x_{\mathrm{cm}}; \quad M^* = \int_h \rho\zeta^2\ \mathrm{d}\zeta = m\rho_g^2. \tag{8}$$

The material density, $\rho$, completely define the material mass characteristics. The mass coefficients become

$$\bar{m} = \rho; \quad m = h\ \rho; \quad \bar{x}_{\mathrm{cm}} = 0; \quad x_{\mathrm{cm}} = 0; \quad m^* = 0; \tag{9}$$

$$\bar{\rho}_g = \frac{1}{\sqrt{12}}; \quad \rho_g = \frac{h}{\sqrt{12}}; \quad M^* = \frac{\rho h^3}{12}. \tag{10}$$

Alternatively, the shell three mass coefficients are explicitly defined as

$$m = h\ \bar{m}; \quad \bar{m} = \frac{1}{h}\int_h \rho\ \mathrm{d}\zeta; \quad m^* = h^2\bar{m}\bar{x}_{\mathrm{cm}}; \tag{11}$$

$$M^* = h^3\bar{m}\bar{\rho}_g. \tag{12}$$