Experimental Measurement of Elastic Shear Modulus of Graphite/Epoxy Tubes

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ABSTRACT

The elastic shear moduli of graphite/epoxy composite materials have been measured using thin walled tubes of different lay-ups. First a dynamic test was used in which shear moduli were derived from the measurement of the natural frequency of torsional vibrations. Next a static torsional test was performed. In both cases the results correlated very well with classical lamination theory. Dynamic moduli were found to be an average 10% higher than static moduli.

EXPERIMENTAL PROCEDURE

Specimens

THE SPECIMENS WERE fabricated from 6, 8 or 12 layers of prepreg wrapped around a mandrel made out of high temperature casting compound (D-Aircraft Product Co.: Dapcocast #38-3). The prepreg was Hercules AS1-3501-6. Inner diameter of the specimens was 76.70 mm with a length of 330 mm. Average ply thickness was .144 mm and the average density of the material, 1473 kg/m^3.

Table 1 shows the stacking sequence of the different layers beginning with the innermost ply.

The 0° plies were aligned with the axis of the tube. As can be seen, the laminates do not possess midplane symmetry because it is not a relevant factor for the torsional stiffness of thin walled cylindrical specimens.

Each attachments consisted of aluminum plugs which fitted tightly inside both ends of the tube. The specimen was then bolted onto this plug through 8 equally spaced bolts and curved washers were used to distribute the clamping pressure. (See Figure 1).

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Table 1

Specimen Type	*Number of Plies*	*Stacking Sequence*
A	6	+45, −45, +45, −45, +45, −45°
B	6	+30, −30, +30, −30, +30, −30°
C	8	0, 0, +45, −45, +45, −45, 0, 0°
D	6	+15, −15, +15, −15, +15, −15°
E	12	0° unidirectional

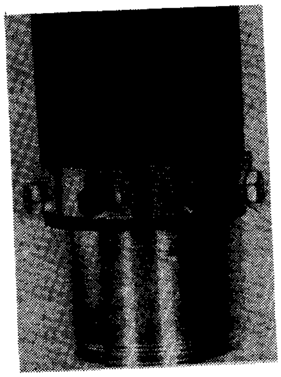

Figure 1. Detail of the end attachment.

Figure 2. Torsional vibration set-up.

Dynamic Test

To excite the specimen in pure torsion around its longitudinal axis, one end of the tube was placed into a bearing. A lever arm was used to convert the longitudinal motion of a shaker into small amplitude torsional vibrations. An inertia mass, large in comparison to the mass of the tube was added at the other end of the specimen to lower the first natural frequency which was detected by means of accelerometers mounted on the inertia mass. The set-up is shown in Figure 2.

The natural frequency has been measured for 6 different amplitudes of vibrations and these results were used to calculate the dynamic shear moduli using a simple analytical prediction of the natural frequency [2]. All testing was accomplished at very small amplitude due to power limit of the shaker (a 100 lbs capacity Ling Electronics shaker was used.). The strain in the specimen was below 50 microstrains.

Static Test

The specimens were tested in a RIEHLE S274, 60,000 inch-pound capacity torsional machine. This equipment provided a torque reading and the shear strain was measured by two ±45° rosette gauges. For specimens of type A, B, C, D, the load strain curves were linear up to failure so that the shear modulus was obtained by a simple linear regression. For type E specimens the load strain curve was slightly non-linear so that the regression only used data points up to half of the failure load.

CORRELATION OF RESULTS

In order to correlate experimental results, it is necessary to derive the shear moduli in terms of the basic unidirectional material constants and the stacking sequence of the plies by means of classical lamination theory.

It can be seen that one of the principal axes of the laminate is parallel to the longitudinal axis of the tube and hence there is no coupling between inplane shearing and inplane extension. The shear force per unit length is thus simply related to the laminate shearing strain by

$$N_{12} = 2 A_{1212}\, \epsilon_{12} \tag{1}$$

where

N_{12} is the resultant shear per unit length.
ϵ_{12} laminate tensor shear strain.

According to classical lamination theory the stiffness coefficient A_{1212} of the laminate is found by summing up the contribution of the different plies,

$$A_{1212} = \sum_{i=1}^{N} t_i E^i_{1212} \tag{2}$$

where

t_i is the thickness of the ith ply,
E^i_{1212} is the shear modulus of the ith ply with respect to the longitudinal axis of the tube, and
N is the total number of plies.

The shear modulus E^i_{1212} depends on the orientation angle θ_i of the ith ply with respect to the longitudinal axis of the tube. It is convenient and informative to relate E^i_{1212} to the invariants as follows [1]:

$$E^i_{1212} = I_2 - R_2 \cos 4\theta_i \tag{3}$$

where I_2 and R_2 are two of the four invariant material properties which are related to the basic four elastic constants of a uni-directional laminate by

$$I_2 = \frac{1}{8D}\left[E_L + (1 - 2\nu_{LT})E_T\right] + \frac{1}{2}G_{LT}$$

$$R_2 = \frac{1}{8D}\left[E_L + (1 - 2\nu_{LT})E_T\right] - \frac{1}{2}G_{LT} \qquad (4)$$

where $D = 1 - \frac{E_T}{E_L}\nu_{LT}^2$

Then using (2) and (3)

$$A_{1212} = \sum_{i=1}^{N} t_i (I_2 - R_2 \cos 4\theta_i) \qquad (5)$$

In terms of the usual shear modulus G given by

$$G = A_{1212}/t \qquad (6)$$

where t is the thickness of the laminate and defining a constant κ depending upon the lay-up

$$\kappa = \sum_{i=1}^{N} \frac{t_i}{t}\cos 4\theta_i \qquad (7)$$

the equation (6) becomes

$$G = I_2 - \kappa R_2 \qquad (8)$$

this relation means that if classical lamination theory holds, the G vs κ plot should be a straight line.

RESULTS

A total of 25 specimens were tested both in torsional vibrations and under static torsion. Average shear moduli measurements are given in Table 2. The numbers in parentheses are the coefficients of variation on the corresponding measurement.

The excellent correlation of these results with classical lamination theory is exhibited in Figure 3, a G vs κ plot, which should be a straight line according to (8). The invariants I_2 and R_2 are determined by means of a standard linear regression analysis on the test data shown in Table 2.

Table 2

Specimen Type	*Number of Specimens*	κ	*Dynamic G (GPa)*	*Static G (GPa)*
A	6	−1.0	32.18 (4%)	29.36 (3%)
B	6	−0.5	28.05 (3%)	23.30 (2%)
C	4	0.0	19.30 (3%)	17.05 (5%)
D	5	0.5	14.05 (1%)	11.80 (3%)
E	4	1.0	6.33 (1%)	4.64 (4%)

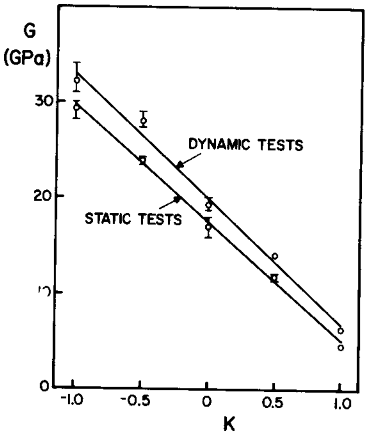

Figure 3. Correlations of test results.

Table 3 lists the values of the invariants for both static and dynamic cases. For comparison the nominal TELAC values are also given [3].

Table 3.

	Dynamic Test	*Static Test*	*TELAC Nominal Values*
I_2 (GPa)	19.98	17.35	19.93
R_2 (GPa)	13.14	12.31	13.93

In the dynamic test the strain level was very low (below 50 microstrain) whereas the entire strain range (up to failure) was covered in the static test. This fact provides a partial explanation for the observed differences between static and dynamic moduli.

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